A MODEL FOR INDIVIDUAL-GROUP COMPARISONS

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1. INTRODUCTION

In problem solving situations, it has been suggested that the superiority of groups over individuals is due simply to the fact that groups consist of several individuals. Here, the Lorge and Solomon [1955] approach to such situations is reexamined using the method of maximum likelihood. Extensions to trichotomous response situations are also presented, and the resulting models are applied to data gathered by Staub [1970]. A more detailed analysis of this set of data, including the partitioning of the likelihood ratio goodness-of-fit statistic, is given in Fienberg and Larntz [1971].

2. THE LORGE-SOLOMON MODEL

For problem solving situations, one criterion for comparing group and individual performance is the difference between the proportion of individuals and the proportion of groups sucessful in the solution of a particular problem. Shaw [1932] compared individuals and groups of size four in just this manner. A group of 41 students was randomly divided into two parts, one part consisting of 5 ad-hoc like-sex four-member groups. The 21 individuals and 5 groups then attempted to solve each of three well-known mathematical puzzles. The data are presented in Table 1.

Lorge and Solomon [1955] suggested that the following hypothesis might provide an adequate description of this data:

> H_0: "Group superiority is a function only of the ability of one or more of its members to solve the problem without taking account of the interpersonal rejection and acceptance of suggestions among its members."

A model for the data, implied by H_0, is described below.

Assume that there are N_I individuals each of which has probability p of not solving a particular problem, and that the number of individuals, n_I, who do not solve the problem is binomially distributed. Similarly, assume that there are N_G groups each of which has probability g of not solving the problem, and that the number of groups, n_G, who do not solve the problem is also binomially distributed.

Under H_0, the probability of a group solution is equal to the probability of the group containing one or more members who can solve the problem, i.e,

$$(2.1) \qquad H_0\text{: } 1-g = 1-p^k,$$

for groups of size k. Thus g is expressible as a function of p under H_0, and the maximum likelihood estimator of p under H_0 is found to be a solution of

$$(2.2) \qquad 0 = (n_I+kn_G)(1-p^k) - (N_I-n_I)(p^k+p^{k-1}+\cdots+p) - k(N_G-n_G)p^k.$$

It can be shown that, under H_0, the Pearson chi-square statistic

$$(2.3) \qquad X^2 = \Sigma\frac{(0-E)^2}{E},$$

and the likelihood ratio chi-square statistic

$$(2.4) \qquad G^2 = 2\,\Sigma\, 0 \log\frac{0}{E},$$

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TABLE 1

Data from Shaw [1932]

Problem I

	I	G_4
NS	18	2
S	3	3
Totals	21	5

Problem II

	I	G_4
NS	21	2
S	0	3
Totals	21	5

Problem III

	I	G_4
NS	19	3
S	2	2
Totals	21	5

TABLE 2

Expected Values and Goodness-of-fit Statistics for the Shaw Data.

Problem I

	I	G_4
NS	17.48	2.40
S	3.52	2.60
Totals	21.00	5.00

$G^2 = 0.226$ ($\alpha = .64$)

$X^2 = 0.306$ ($\alpha = .58$)

Problem II

	I	G_4
NS	19.26	3.54
S	1.74	1.46
Totals	21.00	5.00

$G^2 = 5.663$ ($\alpha = .017$)

$X^2 = 4.823$ ($\alpha = .028$)

Problem III

	I	G_4
NS	18.77	3.19
S	2.23	1.81
Totals	21.00	5.00

$G^2 = 0.059$ ($\alpha = .81$)

$X^2 = 0.070$ ($\alpha = .72$)

are each asymptotically distributed as a χ^2 variable with 1 degree of freedom, where O is the observed value, E the expected value based on the maximem likelihood estimate, and the summation is over all four cells of the 2x2 table. The test statistics X^2 and G^2 are goodness-of-fit statistics used to test the fit of H_0 against unrestricted alternatives.

For the Shaw data in Table 1 the maximum likelihood estimates of p are 0.832, 0.917, and 0.894 for Problems I, II, and III respectively. Table 2 contains the expected values under H_0 for each of the three problems along with the values of the goodness-of-fit statistics G^2 and X^2.

3. EXTENSION OF MODEL TO TRICHOTOMIES

The model discussed in Section 2 can be extended to the case of a trichotomy with comparison of individuals and groups of size two. The extension was suggested by data gathered by Staub [1970] on the development of helping patterns in children. In the Staub experiment, kindergarten, first, second, fourth, and sixth grade children, alone or in same-sex pairs, heard sounds of another child's distress from an adjoining room. One of three responses was recorded for each individual or pair. The three responses were 1) NO HELP, 2) VOLUNTEER HELP, and 3) ACTIVE HELP. An excerpt of the recorded data are given in Table 4 under the "OBSERVED" heading.

Looking at the observations for each grade-sex classification separately, the observed table for any particular grade-sex classification is of the form given by Table 3. There are n_1 individuals who gave no help, n_4 pairs that gave no help, etc.

TABLE 3

Observed Table for a Classification

	Ind	Pairs
NH	n_1	n_4
VH	n_2	n_5
AH	n_3	n_6
	N_I	N_G

Assume that there are N_I individuals and the probability of observing (n_1,n_2,n_3), where $n_1+n_2+n_3 = N_I$, is governed by a multinomial distribution with probabilities (p_1,p_2,p_3), where $p_1+p_2+p_3 = 1$. For pairs, assume that there are N_G pairs and the probability of observing (n_4,n_5,n_6), where $n_4+n_5+n_6 = N_G$, is also multinomial, but with probabilities (g_1,g_2,g_3), where $g_1+g_2+g_3 = 1$.

Assuming that a pair consists of two randomly chosen individuals and that the manner in which pairs act is dominated by the more helpful individual, the probabilities for pairs become

$$(3.1)\qquad H_0:\ g_1 = p_1^2,\quad g_2 = p_2^2 + 2p_1p_2,\quad g_3 = p_3^2 + 2p_1p_3+2p_2p_3.$$

Under hypothesis H_0, the parameters (p_1,p_2,p_3) can be estimated by maximum likelihood and the statistics (2.3) and (2.4) can be used to judge the fit of the model. Solving the maximum likelihood equations yields the estimates

$$(3.2)\qquad \hat{p}_1 = \frac{-n_3 + \sqrt{n_3^2 + 4ac}}{2a},\quad \hat{p}_2 = r\hat{p}_1,\quad \hat{p}_3 = 1 - (1+r)\hat{p}_1$$

TABLE 4

Observed and Expected Values and Goodness-of-fit Statistics for the Staub Data

		Observed I	Observed P	Expected I	Expected P	X^2	G^2
Kindergarten Boys	NH	7	3	5.75	4.14	2.41	3.54
	VH	0	3	1.20	1.91		
	AH	1	2	1.05	1.95		
		8	8	8	8		
Kindergarten Girls	NH	6	5	6.19	4.79	0.83	1.14
	VH	2	2	1.47	2.54		
	AH	0	1	0.34	0.67		
		8	8	8	8		
4th Grade Boys	NH	6	2	4.96	2.69	3.41	4.94
	VH	2	1	1.42	1.77		
	AH	0	4	1.62	2.54		
		8	7	8	7		
4th Grade Girls	NH	2	6	5.38	3.16	11.19	11.25
	VH	1	0	0.38	0.47		
	AH	5	1	2.24	3.37		
		8	7	8	7		
6th Grade Boys	NH	9	5	8.86	5.20	2.40	3.12
	VH	0	2	0.89	1.09		
	AH	2	1	1.25	1.71		
		11	8	11	8		
6th Grade Girls	NH	7	6	6.96	6.06	0.78	1.10
	VH	0	1	0.36	0.64		
	AH	1	1	0.68	1.30		
		8	8	8	8		

where

$$r = \frac{n_2-2n_1-4n_4+S}{2(n_1+2n_4)} , \quad (3.3)$$

$$S = \sqrt{(2n_1+4n_4-n_2)^2+8(n_2+n_5)(n_1+2n_4)} \quad (3.4)$$

$$a = (1+r)[(n_1+2n_4)(1+r) + (n_3+2n_6) + \frac{2n_5}{2+r}(1+r)], \quad (3.5)$$

and

$$c = n_1+2n_4 + \frac{2n_5}{2+r} . \quad (3.6)$$

In Table 4, under the expected heading, the fitted values for the excerpt of the Staub data are given. The X^2 and G^2 goodness-of-fit statistics are also included. Both sets of statistics when compared with a chi-square variable on 2 d.f. indicate that H_0 fits the observed data well, except for fourth grade girls. In fact, fourth grade girls helped more as individuals than they did in pairs.

To examine the effect of grade and sex on the applicability of H_0, one can make use of methods for partitioning likelihood ratio test statistics. Details are given in Fienberg and Larntz [1971].

4. SUMMARY

In the comparison of group versus individual behavior of various sorts, a model of interest is the one which postulates that differences in response can be accounted for simply by the fact that groups consist of several individuals. This model has been applied to two different sets of data in this paper, using the method of maximum likelihood and the standard chi-square goodness-of-fit criteria. Complete details for the analysis of these data are given in Fienberg and Larntz [1971], where alternative models are presented and the small sample behavior of the likelihood ratio goodness-of-fit statistic is discussed.

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